Phys 410
Spring 2013
Lecture #31 Summary
10 April, 2013

We considered the $2n = 2$ −dimensional phase space of a $n = 1$ object falling under the influence of gravity in one dimension. In this case the phase point trajectories are parabolas in phase space: $x = \frac{p^2}{2m^2 g} - \frac{E}{mg}$, where E is the total mechanical energy. Hamilton's equations give $\dot{x} = \frac{\partial \mathcal{H}}{\partial p} = p/m$, and $\dot{p} = -\frac{\partial \mathcal{H}}{\partial x} = mg$. These can be solved to yield $p = p_0 + mgt$, and $x = x_0 + \frac{p_0}{m}t + \frac{1}{2}gt^2$. We took 4 different initial conditions in phase space (the x-p-plane in this case), and found that the trajectories followed from those 4 distinct points did not cross. We also observed that the "phase space volume" enclosed by those 4 points did not change as the system evolved in time. This observation is consistent with Liouville's theorem: the volume enclosed by a surface in phase space does not change as the system evolves under Hamilton's equations. This theorem is important for statistical mechanics of systems in equilibrium. There is a quantum mechanical version of Liouville's theorem that applies to a quantity known as the density matrix.

We next considered the most general motion of systems of particles. We specifically consider rigid bodies, defined as multi-particle objects in which the distance between any two particles never changes as the object moves. We reviewed the center of mass, center of mass momentum, and Newton's second law for the CM. We then considered the angular momentum of a rigid body and found that it decomposes cleanly into the angular momentum of the center of mass (relative to some origin), and the angular momentum relative to the CM. For a rigid body, the only motion it can have relative to the CM is rotation.